Mathematics: Mother of the Universe
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Abstract

A logical explanation is presented describing how our Universe could exist, as we experience it, without using any preexisting physical space, matter or any other seed. Consequences are suggested, along with artifacts we might look for in Nature. In addition, several philosophical questions have simple answers in the context of this explanation.
Summarizing video available here: https://youtu.be/tlJ3f5i6F9M

"… Getting real existence from pure logic is just too much of a conjuring trick. That sort of hat cannot contain rabbits."
- Nicholas Rescher, *The Riddle of Existence*, 1984

1. Introduction

By some definitions, the term "universe" includes not just what we can perceive (even indirectly using tools), but all other places or things, that may not be in any way connected or similar to our sense of our natural surroundings. Instead of using that all-encompassing definition, I define a "universe" (note lower case "u") as everything that is connected via the transitive closure of all information or causal connections. Transitive closure just means that everything that is even indirectly connected is included. For example, the distant galaxies, that we see because of light traveling from them into our telescopes, are considered to be part of our Universe. If in those galaxies they perceive something that is beyond our viewable horizon, those somethings are still part of our Universe. I use a capital "U" in the word Universe when referring to our own universe. Other universes have no visibility or communication with our universe, even indirectly. Our Universe is the one in which we reside and by the above definition; we cannot communicate with any other universes.

The "Big Bang", many worlds (Everett 1957), Harrison's multiverse (Chown 2002, 110-112), Smolin's self-reproducing universes (Chown 2002, 112-114), and other models abound (Rees 2001), proposing to explain how the Universe was created. However, as one works backward, they all assume some sort of pre-existing seed, pre-existing space-time, infinite recursion, fluctuating vacuum, or some creating entity, which the theories don't explain. If someone were to claim that *X* created the Universe, the obvious question would be "what created *X*?" To stop this induction, we need to find an *X*, which needs no creation. Mathematics was early seen by the Greeks as important in the nature of the Universe, but the approach of resting the creation of the Universe entirely on mathematics is treated "with distaste" (Barrow 2000, 286) by philosophers and physicists for thousands of years. What has been missing is a plausible explanation of how mathematics alone could suffice as the seed needed to create our Universe. This explanation would most likely be much like a tautology or self-evident argument because it would not require a pre-existing seed or further antecedent. I propose to describe a conjecture that explains the existence of our Universe in terms of mathematics alone. Furthermore, I have found that this explanation produces plausible, sometimes disturbing, answers for many difficult philosophical questions, and even suggests the possibility of certain physical phenomenon that may be present in the nature of our Universe.

Briefly, the conjecture concludes that our Universe exists in the same manner that a number, such as π (pi), exists. The first objection might be that π is a static entity, just a point on a number line. I will argue that our sense of time, space, matter and energy can all be reduced to such a static number. The entire conjecture consists of several such hurdles of understanding, each of which I will address. More explanation will be given about using mathematics as a starting point, because it is the one "thing" that need not be created to "exist." A simulation of an entire universe can be represented as simply as one result in mathematics. Intelligent life, such as mankind, can evolve within the simulation of a sufficiently rich universe. The universe would seem "real" or "physical" to all of the entities inside the simulation. Physical existence is a perception dependent on the perspective of the observer. The implications are that there are an infinite number of universes and an infinite number of ways for each to "exist." Such a conjecture would be difficult to test for several reasons. First, the conjecture explains the existence of every universe, and thus there would be no counterexample universe that might be inconsistent with the conjecture. Second, we are most probably unable to recreate a sufficiently complete model of our universe within itself, in effect providing a demonstration of a positive proof. However, we are able to observe and experiment with smaller "universes" or tiny portions of our Universe, via computer simulation for example. Further implications do suggest some interesting artifacts in nature that have some probability of being found in our Universe. Finding any of these would support this conjecture, but the lack of them would not be able to invalidate the conjecture.

Several age-old philosophical questions have simple answers in the context of this conjecture. Of all of the possible ways that this Universe might have been designed, why, for example, does it have 3 space dimensions, one time dimension, four types of forces, quantum mechanics, relativity, a small set of elementary particles, and why are various constants such as the gravitational constant, the speed of light, the charge on an electron, or the mass of an electron set exactly at their specific values? And, why is matter and energy distributed in the Universe exactly the way it is, with a Milky-Way galaxy containing a small star about which a blue planet sustains life? As a mathematician, one would say the probability of having our Universe exist rather than the infinity of other possible universe configurations that might be imagined, vanishes to zero. Next, one might wonder whether all possible universes exist or none. Would it not be simpler to build no universes? Yet we are here. The question, "Why is there anything at all?" was asked by Gottfried Leibnitz and many others. Where is our Universe? This paper produces answers to these and other questions. I will explain that in certain senses, and not paradoxically, all of the following are true: 1.Ultimately, there is no physical existence – it just seems physical because we are viewing it from within the Universe, 2. Our Universe exists within mathematics, 3. An infinite number of universes exist, and more.

2. Physics and Simulating the Theory of Everything

A "Holy Grail" of science is to find the "Theory of Everything." This theory would explain how the Universe operates from its beginning, if there is one, to its end, if there is one. Let us assume that this theory specifies a set "*A*" of laws, rules, procedures, and initial conditions, which are needed to explain and predict in principle, if not practice, how everything behaves in our Universe. So far, scientists have discovered many important laws that quite accurately describe this Universe. Examples include the laws of motion in space-time, how four forces of nature seem to affect everything, the equivalence of matter and energy, modifications to "classical" rules per quantum mechanics and per the theory of relativity, and so forth. As we make more detailed measurements of our Universe, the scientific method makes it necessary to revise certain laws to reflect new understandings about what has been observed. A prime example is Einstein's relativity, correcting errors in Newton's laws of motion, most noticeable when relativistic speeds are approached. More refinements and additions to these laws are necessary before we will be satisfied that we have a complete set *A*.

The laws in assumption *A*, that we have found in science so far, might be stated as differential equations, which with suitable initial conditions predict the behavior of a part of the Universe being studied. There appear to be other equivalent mathematical ways to express these laws, for example, using "string theory," "loop quantum gravity," "cellular automatons," or probably many other undiscovered mathematical methods. Some of the laws might not be suitable for formulation as differential equations and may require other techniques. Humans may never discover all of the laws governing our Universe, perhaps because the set is too large, or because some features may not be fully observable by us in a way that permits deducing them with certainty. For the purposes of this conjecture, it is not important how large set *A* is, and suffice it to hypothesize that there is some set of laws, however large, that describe the way this Universe works. For initial simplicity, let us assume there are only a finite set of laws in *A* and that our Universe is finite. I will later show how these restricting assumptions might be relaxed.

A description in terms of differential equations does not lend itself to ready solutions which show how matter and energy behave except in the most simple of cases. For example, we do not have closed form equations that show how three point masses move in space when they interact only with gravity. As soon as we add more forces and particles, the situation becomes hopelessly complicated. Physicists typically make all kinds of assumptions: zero friction, point masses, often consider only the first or second order terms in their equations, and others, in an attempt to simplify the problems to make some sort of study tractable.

The difficulty in exactly solving the equations, which encode the laws of the Universe using conventional mathematical notation, grows immensely with the number of variables (number of particles and size of space) being considered. Scientists turn to simulations to solve problems that are too difficult to solve in closed form. If one is interested in determining how the 3-space position of a particle changes over the time axis, one can start by chopping the time axis into very small time increments. For each increment, the sum of all forces on that particle are calculated, and then the resulting change in velocity is applied to the particle's initial velocity at that time increment, and a new trajectory is computed to find where the particle will be at the end of the time increment. This can be calculated for all particles and points of interest, and repeated until the desired time or future event is reached. The method looks much like integration and is used to calculate trajectories for spacecraft traversing our Solar system and can equally be applied to particles. Especially for small particles, quantum effects must be included in the simulation. A later section will deal with the problems of indeterminism from quantum processes and warping of space-time.

It would be computationally prohibitive for us to compute the exact future of our Universe. If the Universe is about 20 billion light years in diameter, and if the simulation requires 3-space to be divided into cells, which have a size on the order of the Planck length (1.6×10^{-35} meters), then this would give approximately 10^{185} cells, into which the entire space of our Universe would be subdivided. The Planck length, named after Max Planck, is conjectured to be the smallest possible length of space, implying that space may consist of tiny discrete cells. It is about 10^{20} times smaller than a proton, so even for microscopic volumes of space, the task of exact simulation would be enormous. Computers represent numbers as scaled integers with a specific maximum size. These integers would need to represent at least 185 digits of accuracy, and probably much more for a full Universe simulation. With a sufficient number of time step increments, tiny round-off errors accumulate when computing with integers rather than exact values. These accumulated round-off errors can add up over many computation steps and eventually lead to substantial error, sufficient to not show the true behavior of what is being studied in the simulation. To reduce the accumulated round-off errors, more precision is used in the computations, and finer subdivisions are used for the time increments until accuracy sufficient for the scientist's purposes is achieved.

The important point is that with a hypothetical computer much larger than our Universe, and using computation time much longer than the known existence of our Universe, it would be *theoretically* possible to compute the behavior of our Universe with any desired level of accuracy using the laws in assumption *A*. Some may object to the prior claim on the basis of chaotic behavior or laws that specifically introduce random outcomes. I will account for these later. However, to increase accuracy, space-time measurements could be subdivided until the required precision is reached. We might, for example, further subdivide our smallest unit of measure 10^{200}-fold and use integers with at least 10^{200} digits of precision. Obviously we could not even store one of these integers in our own known Universe, but that does not matter, as we shall see later. And, even with this vast increase in precision, we still have a finite description of the Universe under the assumptions we have used thus far.

3. Outline of a Simulation of Our Universe

I will briefly, and superficially, outline how a classical simulation of our Universe might be designed, for those not so familiar with such methods. First, one might devise an appropriate coordinate system. In our Universe there seem to be three space coordinates and one time coordinate, although one could argue that time is not actually a coordinate. String theory suggests that there might be quite a few more short, looped coordinates, but we will ignore those and other complications for simplicity of illustration. The coordinate system is given a minimum measurement increment, on the scale of the Planck length for the x, y, and z axes, and Planck time for the t axis. Then, each of the cells is mapped to a computer memory location. Even though computer memory is essentially linear, any number of dimension arrays can be mapped to this linear memory. For infinite dimensions, various diagonalization techniques can be used to map multiple dimensions to one memory dimension. Thus any number of space dimensions can be mapped to a computer memory.

At each memory location, various values are stored about the cell at that coordinate according to whatever is specified in the rules in *A*, which describe the operation of the universe. For example, values stored might be scalar or vector values for various fields, probability of finding a specified mass and momentum in the cell, and so forth for all other required properties. An additional virtual coordinate *s*, corresponding to the computation steps should be mentioned. At each computation "step," the new values for each cell in the coordinate system are calculated and stored back into the cells. One of the computations might involve calculating the sum of all force fields acting on a particle in a given cell and adjusting its location and momentum accordingly. The computation "time" coordinate *s* may or may not largely correspond to the simulated time *t*, depending on how the rules in *A* are formulated. Time in our universe is really a causality / dependency structure rather than a dimension, but discussion about this will be omitted here.

The initial conditions for the simulation could be set to simulate the Big Bang, by inserting an enormous mass into the coordinate system at the origin, with specific attributes of momentum and other characteristics. As the computation steps proceed along the *s* coordinate, the description of the universe unfolds in the simulated coordinates *x*, *y*, *z*, and *t*, which themselves need to be expanded from a singularity in the simulation, to match the expansion of space.

The procedure outlined here is deterministic. If non-deterministic rules exist in *A*, then these can be simulated as well. For each point at which a rule suggests a range of alternative outcomes, our simulation can proceed by exploring every alternative. The entire contents of the coordinate space are copied and simulation continues from each copy, representing each of the possible alternatives. Of course, as multiple non-deterministic decisions are made on top of others, the number of copies of the coordinate space and its contents grows exponentially. Readers may recognize the similarity of this simulation procedure with that of handling quantum uncertainties using Everett's (1957) many worlds interpretation of quantum mechanics. Briefly, Everett contends that the Universe splits at the moment of measurement, leaving us in one of the split universes to perceive the quantum outcome. More generally, we can use the interpretation that at each moment, the Universe splits into multiple Universes corresponding to all possible combinations of the results of quantum effects. Thus the history of the universe is represented by an enormous tree structure containing all possible paths to the future and exactly one path backward in time from any position. The act of measuring simply identifies the Universe in which the particular copy of the observer resides. The approach explains all quantum weirdness as well as spooky action at a distance. The only objection is that this approach has been considered "wasteful" because of the enormous number of split Universes that the approach requires. However, we will see that mathematics provides the needed infinities, which easily handle this "wasteful" continual splitting of the Universe, as will be shown later. Incidentally, this also explains, in combination with the anthropic principle why our Universe is not completely symmetric. We are simply in one branch of the tree of split universes, each branch breaking symmetry in its own way.

4. Watching a Simulation
Some popular video games are implemented using simulation methods. Most computer games have input and output exchanges with the human player. Human interaction is not always necessary to make an interesting game. For some game simulations, one may set the initial conditions and just watch to see what happens. Many of the "Sim" series of games (SimFarm, SimAnt, SimCity…, trademarks of Electronic Arts Inc.) are of this nature, although there is opportunity for the user to intervene at any time.

Some believe the Universe is a giant cellular automaton and all we need to do is find the rules, which govern it. Actually, a very simple cellular automaton can in turn simulate any other cellular automaton, albeit, inefficiently. James Horton Conway, a remarkably inspired mathematician, invented the "Game of Life," (Gardner 1970) which is a cellular automaton -- a simulation in a tiled, two dimensional space of "full" or "empty" square cells, obeying a set of very simple rules. The birth rule: if a cell is empty and has exactly 3 of its 8 neighboring cells full, then the cell becomes full on the next computation step. The under- and overcrowding rule: if a cell is full, it stays full on the next computation step only if 2 or 3 of the 8 neighboring cells are full. Initial conditions are set by making some cells full and then seeing what happens in successive computation steps. What happens is very interesting and ultimately profound. First, it turns out that the Game of Life is computationally complete. This can be shown by implementing what is called a "Universal Turing Machine" within the Game of Life. A Universal Turing Machine can be shown to be able to compute any kind of algorithmic computation. This means any computation *X* can be translated into a configuration x' in the Game of Life, and the execution of the game will eventually (and quite slowly) produce configuration y' which when translated back produces the result *Y* of computation *X*. This is much like running the same algorithm using different programming languages and computer architectures, yet achieving the same results.

One should study any of a number of web sites devoted to the Game of Life. The reader should find a recently written book by Stephen Wolfram (2002), entitled "A New Kind of Science," very interesting as well. It explores cellular automata of various kinds showing how they can demonstrate the complexities found in nature using remarkably simple rules. The book also makes important observations about the concept of irreducible computations: that the outcome of some computations cannot be predicted with less effort than actually performing the computation.

Our simulation of the entire Universe of 10^{185} space cells, using very large integer values for the coordinates and properties of those cells, and the rules in assumption *A* to be applied to these cells, can all be theoretically represented by a very large Game of Life configuration, which over a very long time shows the behavior of our Universe.

5. Important Observations About Simulations
The first important observation to make is that **simulations are deterministic**. If you start with exactly the same inputs, including same random number generators, providing inputs at exactly the same times relative to the start of the game, and ensuring the same ordering of computer instruction execution [1], you get exactly the same results every time you compute the

simulation. That is, the computation is deterministic. Non-deterministic computations can be computed deterministically but with much greater effort, by computing every combination of the non-deterministic choices.

The second observation is that time t dimension within the simulation is virtual and does not necessarily reflect the simulator's time s, which is really a computation step dimension. **Simulation time *s* is not perceived within the simulation.** We may run the simulation on computers of different speeds, producing exactly the same result, but using different amounts of computation time. Furthermore, if the software is so designed, one can stop the simulation part way, save the intermediate results, and then resume the simulation much later, producing exactly the same result. Within the simulation, we could simulate the concept of multidimensional time and still implement the simulation using a single dimension of computation step time s. Entities inside the simulation are oblivious to pauses in the computation of the simulation. For example, the SimFarm characters are not aware that a child saved the state of their world, and then continued it the next day. A particular simulation can be thought of as a movie film. Each frame captures the state of the simulated world at each computation step. This is easy to imagine when the computation step dimension largely matches the time dimension being simulated, as it might for a simulation of our Universe. At each simulation step, the characters in the simulation are in a state of knowing their history and expecting certain things in their future, yet they would not perceive the simulation as being in steps. If we let this simulation run from the start to the end of time for the simulated Universe, we have a complete description of everything that happens.

The third observation is that entities inside the simulation are affected by each other, and if there is no provision for outside input, they are totally "unaware" of us watching the simulation. **The way reality is perceived by an entity depends on if the entity is in the simulation or not.** For example, if our Universe were a "SimUniverse" game for a some super-giant "child" within a much larger universe, this "child" would see life eventually evolve on the tiny planet that the people living there call "Earth", with those people busy raising their families, doing their work, and so forth. The intelligent "people" within this simulation would think they really feel matter and forces, would think they all "exist", and interact with each other accordingly, yet they would be oblivious to the giant "child" watching their world.

The fourth observation is that a simulation has to be rendered only if someone outside of the simulation wants to perceive it using their perceptive mechanisms of choice, in our case eyes, ears and other sensory organs. **To perceive a simulation from the outside, it must be rendered.** The computer in the prior paragraph simulating the SimUniverse game would be performing its computation steps for every particle and every quantum of energy for that universe without any kind of external rendering. The people and other entities within the simulation would still feel their universe is real. The "child" in the parent universe might have a "display screen" for rendering some interesting portion of the simulation, such as the portion located on the planet Earth, but that would not affect the perception of the simulated universe by the entities within it. However, the mathematical progression of the simulation is static and already determined without actually going through the steps of simulating the simulation. The characters at every simulation step inside the simulation are in a state where they perceive motion, history, and interactions with the other entities within the simulation. Yet, we need to actually implement the simulation only if we want to view it from the outside.

We take the simulation, a mathematical deterministic process, and let that simulation's progress rate along *s* tend to zero, and conclude that even though the simulation thus never gets rendered for an outside observer, it still is perceived as normal and real for the entities within the simulation, because the simulation is rendered for them within mathematics. This means, there does not need to be something executing the simulation for the entities within the simulation to perceive their simulation as "real."

6. Mathematics is Discovered

Mathematics starts with a set of assumed axioms and deals with the results that logically follow starting with those axioms. Various fields of mathematics start with different sets of axioms and rules for manipulating them (logic..) and this leads to the concept of counting, adding, geometry, algebra, calculus, and so forth. There is a story about a person who was placed in a prison, and all by himself, started with simple axioms and discovered his own paths to the same mathematical theorems that others had found on the outside. Mathematics, in this sense, is never invented, just discovered. If a civilization or intelligence is able to understand counting, sets, etc., that civilization will eventually find the same mathematical truths. Even God cannot change the fact that 2 plus 2 is 4, or that 7 is a prime number. These are simply an absolute implication from a set of axioms, definitions, and logical manipulations. If you alter the axioms, definitions, or the logic, you can perhaps say that 2 + 2 is 1 (as in mod 3 arithmetic) but you are then talking about a different set of axioms, definitions, and logical manipulations that produce an equally immutable set of implications.

The number π (in decimal notation 3.141592…), which is the ratio of the circumference of a circle to its diameter, will most likely be discovered by any sufficiently intelligent life, even if they do not live in a mostly Euclidian space like ours. There are an infinite number of equations, which produce π as their solution. The number π exists whether anyone has discovered it yet or not, and whether anyone has written it down in decimal notation or not. In fact, since it is not rational, nobody in a finite universe can write it down in full in any integer radix notation. And, since it is derived from the pure logic of mathematics, its existence is not dependent on the existence of this or any other universe. Anyone sufficiently intelligent will stumble upon the concept of the number π as well as many other interesting things in mathematics.

Simulations are computations which can be represented and computed on a simpler form of computer called a “Turing machine.” Thus any algorithm implementing any simulation can be translated into a program that executes on a Turing machine. It has further been shown that any Turing machine program can be represented as a diophantine equation (Chaitin 1999). A diophantine equation is an algebraic polynomial with the requirement that the solutions be integers. The process of finding an integral solution to an appropriately written diophantine equation can be equivalent to computing a Turing machine program. Thus, the computation of a simulation can be represented as the solution of a diophantine equation.

7. If π exists, then so does our Universe

The point of the above is that a simulation of a universe can be represented as the solution to a single equation in mathematics. Similarly π can be represented as a solution to a single equation in mathematics. I claim that our Universe exists exactly in the manner that the number π does.

The simulation of our Universe could be one (quite large) solution to the diophantine equation, which plays out the solution we are living, if the solution were to be rendered. Note that if we are ***in*** the simulation, we don't need to stand back and see it from the outside for it to seem real to us. Our perception of something as real or physically existing is relative to our point of observation. We feel that our Universe physically exists because we are in it, and it only seems to us that the Universe's physical existence is necessary for ourselves to exist. It does not matter that no physical computer is computing the answer. We don't need to have the simulation rendered. Neither do we need to write down all of the digits of π for it to exist. Nobody needs to render our Universe for it to exist. Our Universe is just a very interesting part of mathematics. Outside of our Universe, nothing has to exist. No space, no time, no matter, no energy etc. Nothing physical needs to be created for our Universe to exist. We just happen to have (obviously) discovered our own Universe because we are in it. We have evolved to the point where we are self-aware, intelligent, and now, are possibly aware of the nature of our own existence.

Next, I will discuss several implications of the above, and in light of this conjecture, answer various philosophical questions accordingly. I will also discuss how to relax the assumptions made earlier.

8. Infinite number of universes

If our Universe is a solution to a hypothetical set of equations or laws, certainly there are an infinite number of these. This would imply that a vast infinity of universes exist, in just the same way as ours does. We will never communicate with them and can only conjecture about their nature. However, we can render other simple universes within ours by using computer simulations, and intercede in those simulations, to communicate with the entities within, e.g. interceding in a Sim game. For example, everything from the past to the future of this Universe out to infinity or its limits, I consider one universe. If two universes can communicate with each other, then I consider them part of the same universe. The Everett many worlds interpretation is still one universe by my definition since all of the alternative paths are informationally (via causality) connected at their root origins.

Max Tegmark's (1998) paper in many ways is very close to proposing the conjecture in this paper. He struggles with the distinction between physical existence and mathematical existence. When one realizes that physical existence is relative to the observer, and not an absolute property, the difficulties disappear. He also looks for a statistical way to make the conjecture testable. This assumes our "physically existing" Universe is in some way preferred over the set of all universes. Thus the argument succumbs to the human desire to view our home as the center of everything. I claim the conjecture explains the existence of all universes, and thus is not testable.

9. The rules in assumption *A* may require infinite computation

I have assumed that a finite simulation is sufficient to describe our Universe. Even if every point in the finite Universe is affected by every other one, this represents a finite (but large) number of computations. What if we relaxed the assumption about a finite universe? Space may be continuous rather than quantized. The set of rules in assumption *A* may be infinite. Perhaps infinite precision or infinite computation is required. If this Universe is the solution to such a hypothetically infinite mathematics problem, then that does not matter. Nobody has to write down or render the solution for the solution to exist. Nobody has to write down the problem for the problem to exist. We don't have to actually evaluate the infinite number of terms in a series giving π as an answer for π to exist. Only in mathematics can we deal with infinite problems and solutions. If we find that our Universe takes infinite computation to follow rules *A*, this may be more evidence that the universe is simply a part of mathematics. We should take up the challenge to further investigate infinite algorithms and infinite Turing machines, rather than dismiss them as intractable.

The rules in assumption *A* may in fact be an infinite set of rules and use infinitely complicated algorithms. Some mathematically undecidable problems may be decidable if you have infinite time to find the answer. It may be that some features of our Universe might seem to take infinite computation to resolve, yet a mathematics based universe would have no difficulty with these. If this were the case for our Universe, it would be evidence that our Universe is in fact a hypothetical solution to some problem defined by the rules in assumption *A*, while it would appear to be impossible for us inside the simulation to understand.

10. The rules in assumption *A* may not require infinite computation

There is a conjecture that space itself is quantized at the Planck scale. If this is so, then it might look like space in a cellular automaton. Loop quantum gravity proposes a structure with a resemblance to a cellular automaton. In addition to rules defining the interactions among the cells, there may be rules for creating and destroying cells in the automaton space, which would account for the expansion, contraction and bending of space-time. Edward Fredkin (1990) and others conjecture that our Universe *is* in fact a cellular automaton. If our Universe is in fact described as a cellular automaton, then the limit to the speed of communication, that of the speed of light, may be evidence that there is finite computation involved in the simulation of our Universe, at least locally. In the Game of Life, the maximum possible speed of communication is one cell per time step and corresponds to the "speed of light" in the game's cellular space.

Similarly, our Universe may be driven by rules that limit the influence on each tiny Planck length sized cell to that transmitted to it by it's immediate neighboring cells within one simulation step. Long distance forces, which influence any cell, may be transmitted by altering the attributes of cells, one neighboring cell at a time, until the influence reaches the destination cells. This would be consistent with the conjecture that the minimum time increment in our Universe is approximately the Planck time.

We may find that experiments with quantum computing reveal that there is a fixed amount of computation required to explain the quantum world at the smallest scales. If the rules in *A* are a finite set, are local, and operate like a cellular automaton, requiring finite computation per cell, and if space and time have finite extent, then the computation representing our Universe could also be finite.

11. Looking for Evidence

If mathematics runs this Universe, one might look for unexpected evidence of this. Since computation time required to render the Universe is of no concern to those inside a simulation of the Universe, it may very well be possible that infinite calculation is required at every point in our Universe to fully obey its laws in assumption *A*. Thus, we might try to devise an experiment to tap into the result of an infinite calculation in the workings of the Universe. If we could set up an experiment that would compute a result that should take an infinite amount of time to compute, we may find something that appears to be a mathematical oracle. Quantum computing may be on the verge of just such a discovery. We may be able to tap into the mathematics that "runs" our Universe and compute something that should take an infinite number of steps in a classical computer.

Another approach is to tap into the possibly infinite precision behind the mathematics that runs our Universe. Consider how accurately photons can travel across most of the known Universe and still converge properly in phase at our telescopes. There may be problems in actually finding an oracle based on infinite precision because our measurement accuracy may be constrained by the uncertainty principle and we may not be able to cheat our way around this constraint.

12. Assumption *A* may consist of an infinite set of rules, including ones which have not been used yet in our Universe.

Another consequence could be that there is a hidden law that says after *N* years of our time, *X* happens. *X* might be some arbitrary rule such as "any proton in a particular cubic meter of space *Y* will be turned into a photon." This would look like a momentary violation of physical laws to any scientist observing the event. Scientists have not, to my knowledge, encountered such anomalies in our Universe, unless each quantum probability decision is such a law. *X* might be a doomsday scenario, or a reset back to the big-bang, or what looks like an "act of God." Although all of these might be possible in any universe, it is quite improbable in our Universe. Life, as we know it, would probably have experienced many more severe extinctions if such events happened often. Such apparent violations of the steady rules would most likely create such chaos in the operation of our Universe that there would be too little stability for life to evolve as far as it has on Earth. If one goes back to the game of life example, one sees how a very tiny perturbation, such as adding a single dot or removing a dot, usually ends up destroying all structures around the dot, unless the dot is somehow isolated.

At this point it might be worth noting that science tries to develop theories that could predict the future. However, there is never any absolute guarantee that any conjecture will be obeyed in the future, even if it seems to have been obeyed on all occasions in the past. Similarly, we can never be certain that some previously unused rule *X* may come into play. However, the probability of this occurring becomes smaller as the consistency of a universe is repeatedly demonstrated. Our Universe appears to be one of the lucky ones, which have seemingly stable rules in *A*, which seem to have been conducive to the evolution of life on Earth.

At the other extreme, assumption *A* may contain an infinite number of rules that very explicitly state in fine detail how every particle in the Universe should move and behave over all time, rather than specifying a small set of physics laws. This would be analogous to a very detailed movie script of our entire Universe, sitting on some infinite giant's shelf. Such a universe would most probably appear to be governed by seemingly random rules, and seem non-deterministic. However, for each algorithmic rule based universe, there exists a scripted universe that produces the same result. Perhaps our Universe is a combination of algorithmic rules in *A* for large-scale events and scripted rules for small-scale events. Together these could account for the large-scale order in the Universe as well as the small-scale quantum randomness.

13. There are an infinite number of ways to describe each universe

There are usually many ways to formulate a mathematics problem. In the same way, there are many ways to formulate a universe. A universe could be a direct simulation of some laws in *A* of the universe's physics, or a simulation of a cellular automaton, programmed to follow the laws in *A*. One of the easiest to consider is using recursive simulations. No matter what computation a simulation performs, it is always possible to form a larger simulation, simulating the first simulation. This can be repeated *ad infinitum*, giving an infinite number of ways to simulate the same universe. Our Universe may be a toy simulation within a much larger universe, itself a simulation containing a giant child watching a rendering of our Universe. One can use recursion with Turing machines, programming a universal Turing machine within another universal Turing machine. They are all just computations, or equivalently solutions to some mathematics equations. The challenge for scientists is to find the *simplest* set of rules, which formulate our particular Universe. Wolfram (2002) has been searching the space of simple cellular automata looking for ones that might match the workings of our Universe. Once you reach a sufficiently complex cellular automaton that can implement a Universal Turing Machine, then you need go no further because that automaton can compute the computations of all other cellular automatons. Chaitin (1999) concludes that we cannot know if we have found the simplest mathematical representation. Therefore, if we find a set of rules *A* for our Universe, we cannot know if this is the simplest set possible. The conjecture described here, explains the existence of this Universe and all others, but it may not be the only way, nor the simplest way.

14: Why is the Universe like it is?

The anthropic principle explains why our Universe is the way it is. We are simply *in* this one and on this quantum split branch per the Everett interpretation. A consequence of the conjecture I have presented is that an infinite number of universes exist in exactly the same way as ours does. Basically, we are very lucky to be in a universe where the laws governing it are stable over many billions of years and light years, and to be in a universe in which the particular laws governing it offer the opportunity for life to eventually evolve over those billions of years, on a suitable planet like Earth.

Evolution can be defined simply as "that which is best suited to survive, does." Using this definition, it is essentially a tautology. The forward direction of time is really set by evolution along a space-time dimension. The second law of thermodynamics drives everything, on average, to higher entropy. The exception is life, which can be defined as that which consumes energy to decrease entropy. In our Universe, the first things to evolve seem to have been the subatomic particles. They quite quickly evolved to form stable configurations of atomic nuclei. These further evolved to more massive forms giving atoms of heavier elements through the actions of stars. Atoms combined to form molecules. Under the correct conditions, atoms formed amino acids and other building blocks of primitive life. The building blocks eventually produced molecular machinery for self-duplication. Information copying was born. This eventually evolved into the life forms we know today based on the information packets called DNA. DNA is the information needed to construct beings, which consume energy and replicate the DNA. Evolution continues as massive organizations of people become more dependent on the organization and through greater communication and technological advances will eventually take the evolutionary process up a notch when it becomes possible to copy and enhance intelligence. The life forms able to copy intelligence will advance much more rapidly using intelligence to drive evolution, and will probably think of us humans as we do our pets.

On Earth, life as we know it is doomed to not last more than several billion years. The star driving our life will eventually expand, baking us and then cool down, freezing any remaining life. It seems that it is neither energetically possible nor economical for humans to travel to other stars. Our evolutionary successors may be able to, however.

This is all possible because the rules in assumption *A* create a rich environment conducive to this kind of evolution. It is easy to come up with rules that do nothing interesting. Intelligent beings, like us, could not exist in an uninteresting universe, nor be there to contemplate our own existence.

15: Is there free will?

Daniel Dennet (2003) and many others have written volumes on the subject of "free will" or indeterminism. When we realize that the universe is probably deterministic, people often cannot reconcile that with their need to think they have "free will," and are usually quite disturbed by the thought that decisions they make are mathematically determined in advance. The problem lies in the ability to ask undecidable questions and pose irreducible problems, features which appear in sufficiently interesting mathematical systems.

Consciousness is not yet fully understood. However, our decisions are based on our environmental (including our body) inputs, our memories, possibly and controversially quantum events, and our brains ability to process these and act or not act on them. Our consciousness is the additional ability for us to observe ourselves live and make our decisions, to the extent we can. Our observation of ourselves contributes to the decision process. The process is sufficiently complex that it is often irreducible and unpredictable by us, short of just seeing what we do. The decisions feel like they are being made by us and they are, per the rules that govern the physics of our brains, bodies, and our surroundings. We have a feedback mechanism which lets us observe what we decide, reverse our decision, reverse it again, continue this until we tire or choose something seemingly at random, just to disprove a prediction being made by another.

"Free will" is actually an ill-defined concept. There is no experiment that can be performed to test for "free will." Is "free will" the ability to make random decisions? If so, are we really making a decision or just throwing dice or obeying a quantum event and letting something else make the decision? Would we ever knowingly overdraw our credit card if we knew the consequences were much worse than our current situation? We might, if we thought we could get away with it or if we are in a state of mind in which we don't care about the future. Even though we think we have the free will to overspend our credit card, would we actually do it? What if the act were something much more severe or fatal? What if the act were to decide whether to put our pencil down on the left side or right side of the table? Would we take the high road or the low road? In all cases, we come to some decision about what to do or not do. The truth is that what we will do is not totally predictable by others or us. Only a complete simulation of our bodies, our brains, and our environment, considering the smallest possible influencing factors, could predict what we would do. This, of course, would be a totally impossible task for us and thus we are likely to describe unpredictable behavior as "free will."

Some argue that free will is an illusion or a lack of understanding how we think. It is difficult to test to see if a person would make the same decision every time in a certain circumstance because part of the circumstance is your memory of what you chose before. Using severe Alzheimer's patients is the closest we can come to testing this. These patients do seem to exhibit repeated behavior suggesting there might not be any randomness to our "free will." Are we in fact able to make any decision totally without evaluating any prior knowledge, the current situation, and external inputs?

Except for the seemingly random probabilities involved in quantum events, the Universe seems to be quite deterministic. Let's suppose our brains did possess the ability to detect quantum events and to use them to make seemingly random decisions. Hugh Everett (1957) suggests that our Universe splits into separate universes at every probabilistic quantum decision point. This would be multiplied by every possible quantum event at every point in the Universe. This creates large and possibly infinite number of branches from every space-time point. We happen to be here together on this path of branches back to the start, if there is a start. However, from every instant on, copies of us diverge into different universes where the consequences of different quantum events are played out. If our brains use quantum events to decide things, there will be one set of copies of us that go off on one set of universe branches that had us taking the high road and another set of copies taking the low road. The branches may be forever separated so we would never know what happens to those copies of us taking the other decision paths. So it may be that *all* decisions are being taken somewhere, but we just happen to be the result of an infinity of particular prior decisions and random quantum event outcomes.

There may be unobservable factors driving this Universe. The characters in a simulation game may not have any tools or access to determine on what kind of computer their simulation is running. Similarly, we only have the particles we understand to work with, to probe the workings of our Universe. Imagine if we were weightless in a region of outer space in which there were no light, just Ping-Pong balls. And, imagine if we ourselves were constructed out of Ping-Pong balls. Our only method of detecting another Ping-Pong ball is to throw one and catch it on a bounce. That would be very analogous to us probing particles in our Universe using other particles. Our accuracy would be roughly limited to the size of the ping-pong balls. We may never be able to tell that the Ping-Pong balls are hollow, or might never tell what they are made out of because we could not reach energies required to break them into pieces. This is why physicists need to build more and more powerful particle accelerators. They need to break their Ping-Pong balls before they can understand what they are and that there may be a limit to how finely subdivided a Ping-Pong ball or particle can be. If these unobservable factors introduce randomness in our choices, some might perceive this as "free will."

16. How can a mathematics equation build something as rich and beautiful as this Universe?

The number π is quite complicated. Any radix notation of π does not repeat and it is presumed to be normal (digits are randomly distributed). In it you can find all of the works of Shakespeare (and all books for that matter) coded as ASCII computer characters, an infinite number of times.

You can find all nonsense encoded in it as well. A very simple concept of dividing the circumference of a circle by its diameter produces this interesting number.

However, a list of integers will eventually reveal the works of Shakespeare too. There are many ways to start with simple rules and produce extreme complexity. This can be demonstrated by computer programs, cellular automata, and chaotic systems, just to name a few. For example, the Mandelbrot (1983) set is one such interesting example. The set consists of all of the points z_0 in a complex plane which never diverge [$|z_n| \leq 2 \forall n$] with repeated application of the formula: $z_{n+1} = z_n^2 - 1$. This very simple equation implemented as a simple computer program can draw the intricate and beautiful "hillsides" just outside the Mandelbrot set. These hillsides can be drawn by drawing points with a color and altitude as a function of how many iterations of the formula it took to discover the point was not in the set. The Mandelbrot set is symmetrical about the horizontal axis, yet infinitely complicated as one looks at it in greater and greater detail at the edges. It produces beautiful pictures with infinite variations, yet repeating fractal similarities. As Wolfram notes, such computations are irreducible. There is no shorter way to render the results and that is why they are complicated and seemingly unpredictable to us.

What I am saying is that what we see here, as our Universe, is a tiny part of a very complex mathematical computation, a simulation of our Universe, which is much like the fractal Mandelbrot set, but much more rich, complicated, interesting, and beautiful. We exist in some far corner of mathematics that we may never fully understand. And, there are other universes, in farther corners, perhaps many times more interesting than ours.

The Universe is beautiful to us because we have evolved in this one to best fit with our environment on Earth. This Universe is important to us. It is built on a set of laws that permitted a rich evolution of life on Earth in this galaxy and most probably in many other planets and galaxies. We are important to our family and friends and we have evolved to thrive here, for the time being. Other universes are difficult for us to imagine. The size of the computation required to render our Universe is difficult to imagine. 10^{800} computation time steps may be sufficient to render our Universe up to this point in time, but even this is a small number compared to infinity. We have no conception of how large and powerful the infinity of mathematics really is. We might say, mathematics is not just the "Queen of Science," it is the "Mother of All Universes."

People have a difficult time appreciating the magnitude of infinity. Despite overwhelming evidence supporting the theory of evolution, many people cannot fathom the large number of steps, via trial and error over the Earth's approximately 4 billion year history, required to evolve intelligent humans. Moreover, we must have a more difficult time understanding numbers like 10^{2000} and greater, when considering a many worlds simulation, that may be involved in describing this Universe. Yet, these numbers are still finite and small compared to infinity. Nature may be going much farther, perhaps all the way to infinity, to run the Universe.

17. "Why is there something instead of nothing?" – Leibniz

With a mathematical existence argument, an infinite number of universes cannot help but exist. And at the same time, there need be no ultimate physical existence of anything, in the sense that Leibniz considered. However, understanding the arguments in this paper, we realize we can achieve what feels like physical existence with purely mathematical existence, and avoid the problem of having to explain the physical existence of just one Universe.

18. Conclusion

The conjecture I have presented is based essentially on a set of self-evident arguments:

1. Mathematics exists independent of any universe or discovering practitioner.
2. Mathematics can describe an entire universe via a simulation, or a mathematical simulation is a universe.
3. Viewpoint matters: The description of a universe is static to an outside observer, but may be dynamic to an inside observer (also depending on the kind of universe).
4. A causal structure in a universe can create the illusions of motion and time.
5. Evolution at all levels can create very interesting emergent results, such as intelligent life, in a sufficiently rich universe with relatively stable properties.
6. The multiplicative quantum results and anthropic principle explain our Universe's lack of physical symmetry.
7. There are an infinite number of universes within mathematics. In fact all equations can be thought of as universes, but many are uninteresting.
8. All describable universes exist somewhere within mathematics.

FOOTNOTE

1. Achieving determinism with a conventional computer would require compensating for any timing considerations such as the spinning of disk platters, seeking of disk heads, timing of external interrupts and any other asynchronous events.

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